Eleanor Palmer Progression Through Calculation

This calculation guidance contains the written and mental maths methods that will be taught at Eleanor Palmer, ensuring progression towards efficient, accurate calculation underpinned by deep understanding of number. To achieve this we believe:

1. Children must be taught **conceptually and developmentally**, not just procedurally. Formal compact methods of calculation are our goal but we value and teach understanding of how and why these methods work.
2. **Number facts** must be taught and rehearsed throughout the school, in every year group, and quick recall is expected so that fluency is achieved. This will mean that children have no barriers when doing more complex calculations.
3. **Mental calculation strategies** must be taught explicitly in every year group so that children assess each calculation before choosing a method: mental, using informal jottings, or using a formal compact method.
4. Teachers must take time with the children to rehearse and learn number facts and mental strategies which will be **prerequisite** to using the written method being taught**.**
5. **Models, images and expanded methods** must be used to support children's understanding of learning new methods of calculation. **Links** must be explicitly made between these and formal, compact methods. If done at the right stage, children will move swiftly to compact methods; showing the links will support those not quite ready yet, deepen the understanding of all children and keep the whole class together.
6. Learning will stick if children find it fun and novel. We use precise mathematical vocabulary and we also give some of our facts and methods **silly names** to aid recall! For example, wakey wakey, rise and shine, 7 sevens are 49!
7. **Purposeful practice** of new methods of calculation, as well as application in new contexts, is essential. *Practice makes permanent* but ensure each calculation makes a link with the previous one, builds upon it or will draw attention to a difficult point.
8. Teachers will carefully design calculations bearing in mind the **micro-steps** and **misconceptions** thrown up by using different digits. No computer generated worksheets please - only calculations from high quality sources (see maths planning maps for guidance on this, and in our maths policy).
9. Encourage and model writing equations **horizontally** so the children really read and understand the numbers before choosing a method.
10. **Inverse relationships** should be explored and they are great ways to check calculations. Some models such as bar modelling or arrays particularly lend themselves to showing both addition and subtraction, or division and multiplication!

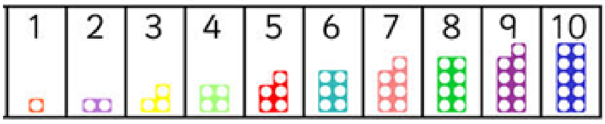
The progression in this policy is **not** set out in year groups. Children should be building on previously taught methods, while the new concepts are becoming embedded. Children **should not be stopped from going on to the next stage** and the pace they move through the stages must be spot on so no-one is held back. Equally, no-one should be rushed through before they have understood conceptually. Our teachers should master this whole document to pitch the correct methods, models and images at the right stage and pace

The models and images set out below are not exhaustive and there will be new, interesting models and images produced from sources such as NCETM and White Rose

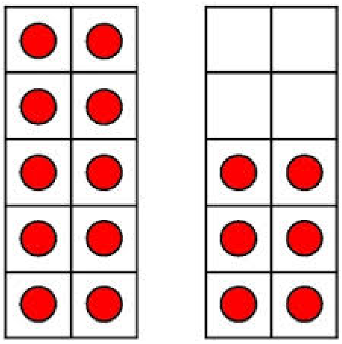
Maths in the future. However, this guidance shows progression in basic terms; please add different models and images judiciously considering how each one will draw attention to and reveal the structure of the maths you're teaching. Remember our maths lead and your colleagues are always open to discussing what works well and to learning new things too!

*It will be useful for teachers to refer to the **Reasoning Think Piece** produced by Camden teachers in a Joint Practice Development Group. It gives suggestions for deepening reasoning when teaching calculation. Please ask for a hard copy in the office! We also have a stage by stage **bar modelling progression document**, which matches calculation objectives with example bar models and advice. Please ask for a copy!*

Models & Images Used at EP

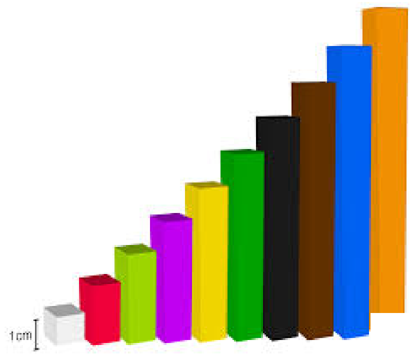


Numicon is used, mostly in EYFS and KS1, to help children understand the 'story' of each number, how much it is worth and to order and compare numbers.

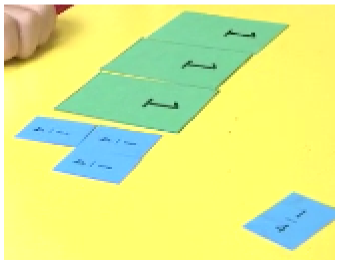


Ten Frames are used, particularly in Reception and Year 1, to help children to subitise (instantly recognise a number) and to begin to understand place value. They can be used in conjunction with dice and counters.

Dice and dominoes are central to all the number games we play at EP. They are essential to developing subitising and to quick factual recall.

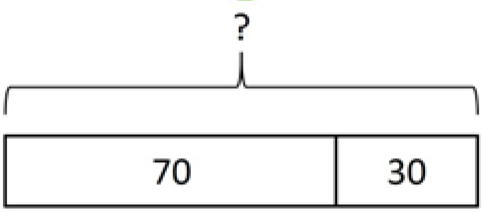


Cuisenaire rods can be used in all sorts of ways but for calculation, they are useful for counting in 'groups of', to learn about number bonds and to form a concrete representation of bar models showing addition and subtraction facts.



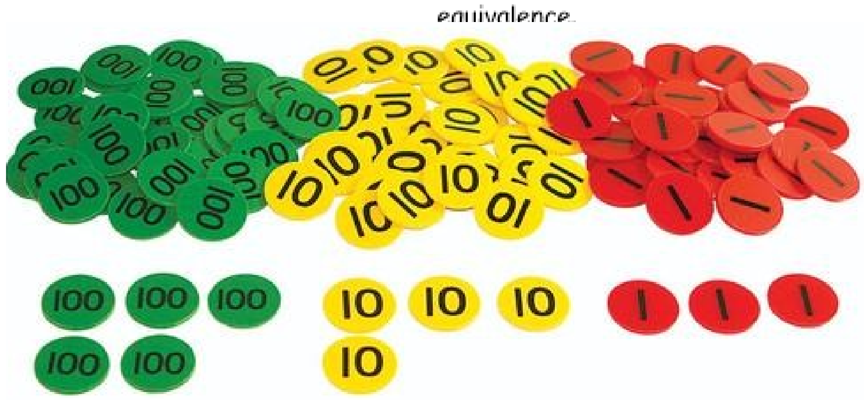
We use fraction cards, organised into fraction families, to help children to add and subtract fractions, and to see equivalence.

Dienes apparatus or Base 10 Blocks are used to represent numbers in calculations, and are effective for helping children to understand place value and numbers' relative sizes.



70 + 30 = 100

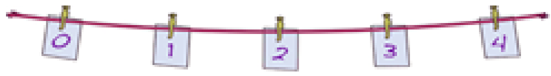
Teachers at EP have begun to use the Singapore Bar Model as another representation to help children calculate with all four operations, and to see ratios and fractions in word problems.



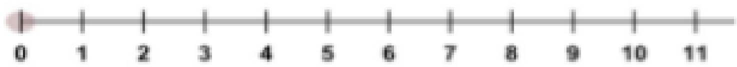
Once children understand the value of different digits, place value counters are a wonderful way to use concrete apparatus to represent written calculations. We use them a lot in Y2 and lower KS2.

This is the order in which number tracks and lines should be introduced and used:

Number tracks and numbers hanging on washing lines:



Number lines with integers labelled. This is a jump conceptually because the number line shows intermediate points. 2 .5 could hypothetically be shown here but not on a number track!



Landmark number lines with 5s or 10s labelled. We call multiples of 10 **Cafe Numbers** and numbers just before or after them, like 19 or 21 are called **nearly numbers!**

Landmark number lines with 10s - cafe numbers - labelled.

Unmarked number lines

Empty number lines.

When the chidren can use number lines, be surprising in your teaching: number lines **don't have to start at 0** and they **don't need to be horizontal**. They could be vertical, diagonal or spiral!

Bead strings are used as a physical representation of a number line. They are useful for all four operations, but particularly for counting on from a number.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The 100 square is formed by cutting up a number track. It is great for counting on in 'wormy ones' and 'spider tens' once children can count on or back confidently using number lines. This is expected towards the end of Year 1 and in Year 2

We use the counting stick to both memorise facts (such as times tables) and to count in different steps. The counting stick is another representation of a number line used daily.

PROGRESSION THROUGH ADDITION

Mental Maths Supporting this Stage

Children must develop a mental picture of the number system to use for calculation. This will only happen after repeated exposure to visual images and models i.e. number line, number track, 100 square, cubes, small world toys etc. Through playing games, singing and rhyming, children progress through the following stages:

- Say number names in order from 0-10 and back, then in Reception, from 0-20 and back.
- Develop one to one correspondence as they count ten objects accurately, then in Reception, 20 objects.
- Count items separately "1..1,2,3,4 – 4"
- Count all the items (without repeats) "1 2 3 4 5 – 5!" and recognise cardinality… 5 is the total!
- Find one more than a number by counting on (hopping on a number track, for example)

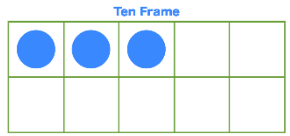
Factual Fluency

- Subitise - this means 'to perceive the number of groups at a glance' ("5!")
- Use ten frames and dice with uncommon arrays, 1-9 dice and dice with numbers can be used as part of progression
- Know the 'story of' numbers up to 5 then 10… number bonds!

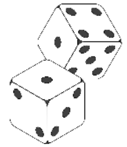
Mental Strategies

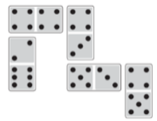
Models, Images & Written Methods

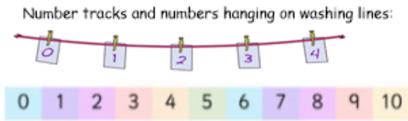
Play with and use Numicon, number tracks then lines, dice, dominoes, ten frames and any

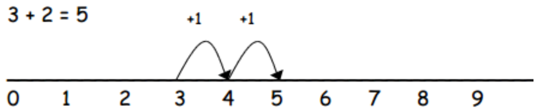


items you can count individually:

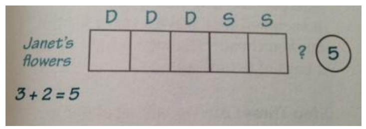




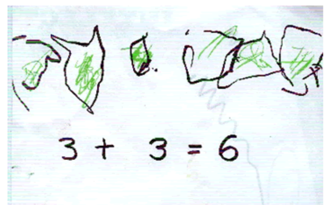




Bar models where one square represents one object can be introduced when children are ready to understand that a symbol or drawing can represent something else. This shows 3 daisies + 2 sunflowers:

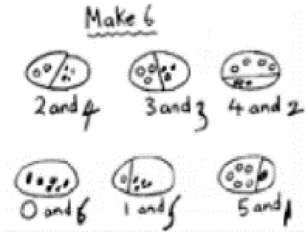


In Reception, children should compare quantities, and using inequality symbols (< and >) as well as = should be taught before + and -. Numerals and the + - and = symbols should be taught and recognised by the end of Reception, and children should progress through drawing and copying to writing their own symbols. These are principles of children's mathematical graphics we must OWL:



Children should always be encouraged to use their own *mathematical graphics* to explore, make and communicate *mathematical* meaning.

- They represent their mathematical thinking in their own ways

• Count on from the larger number "4 – 5!" Put the biggest number first then count on. • Count out a number of objects from a larger group. This is an important and big step! • Relate addition to combining two groups of objects • Find the total of two groups of objects by counting.	• They build on what they already know about the meaning-potential of marks and visual representations to also carry *mathematical* meanings • They co-construct understanding about mathematical notation through collaborative dialogue • They focus on *processes* of learning, rather than notation as a product 

PROGRESSION THROUGH ADDITION

Mental Maths Supporting this Stage

Factual Fluency

- Learn and develop fast recall of number pairs with a total of 10 and 20.
- Use number bonds to add e.g. 23 + 2 = 25 because I know 3 + 2 = 5.
- Develop fast addition of any single digit number to or from a multiple of 10.
- Double all numbers to 20 e.g. 8 + 8

Note: At EP in Y1, Y2 and Y3, we do lots of 'special quizzes' made up of addition and subtraction questions done in a time limit. Ask for copies and examples!

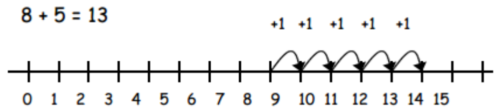
Mental Strategies

- Add near doubles. e.g. 6 + 7 moving on to larger numbers: 20 + 21
- Count up and back to and across 100, or from any given number in hundreds, tens or ones (wormy ones and spider 10s).
- Count on in multiples of 10 and then add nearly numbers (e.g. +11 or +9) by spider counting then adjusting.
- Add any pairs of numbers below 20, moving on to adding trios of numbers and keeping a cumulative total. Number lines can be used to keep track of this adding. e.g. in the game 'Don't roll a 6!)
- Spot nearly numbers then adding/adjusting.
- Explore the law of commutativity so that children can re-order additions, starting with the biggest number: 5 + 48 = 48 + 5.
- Record and complete equations, understanding the = symbol as showing equivalence rather than 'is the answer'. e.g. 20 = 15 + ∆

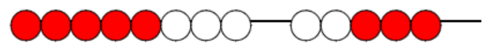
Try some other equations which require balancing:20 + ∆ = 23 + 2
Add two, three or more coins by regrouping coins of the same denomination.

Models, Images & Written Methods

- Children use number tracks then number lines to count on in 1s. Teachers must demonstrate this!

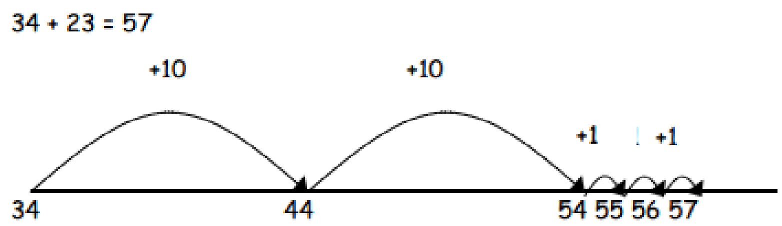


- Bead strings or bead bars can be used to illustrate addition, including bridging through ten by counting on 2 then 3.



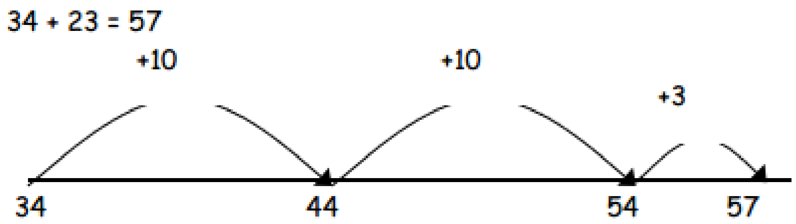
- In Year 1, children should be given teacher's modelling and scaffolding to start to record their calculations, as well as their answers. This might take the form of a landmarked number line or a number track, for example. In Year 2, children should begin to record their addition using number lines independently, moving towards an empty number line.

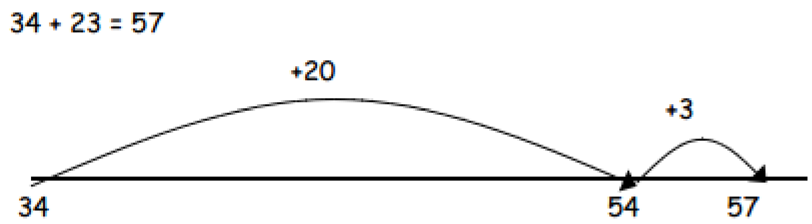
- To add two digit numbers, children should start to use empty number lines themselves to partition numbers and add tens then ones, recording this in their books:



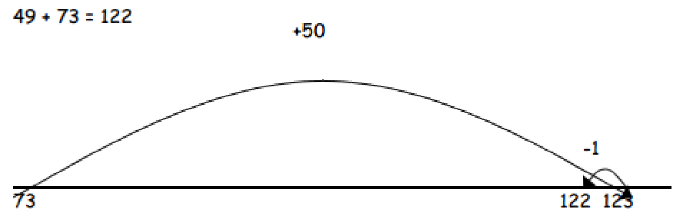
PROGRESSION THROUGH ADDITION: Models, Images & Written Methods

- Next children should use known facts to add the ones in one jump (by using the known fact 4 + 3 = 7) then the tens in one jump:





- Adding nearly numbers such as 49 (nearly 50) then adjusting by subtracting 1, can be clearly modelled using a number line. Informal recording of this mental strategy should be encouraged:

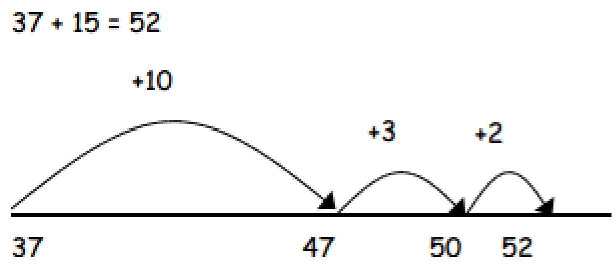


- Number lines are key images for adding hours and minutes, and pounds and pence, bridging through hours or pounds.
- Model counting in **wormy ones** on the number line and then, when children are secure, on the 100 square. Then count on in 10s. We call this **spider counting**.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

This is often trickier for the children than adding on a number line, but it is an important image for the children to become familiar with so they can add nearly numbers and bridge 10 on a number line (see below)

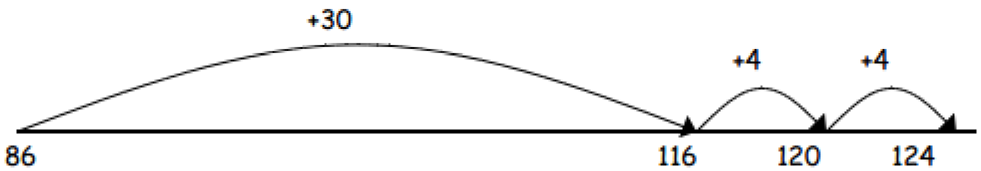
- Once this is mastered, add nearly numbers e.g. + 11 by adding 10 then 1. This will make an L shape on the 100 square.
- Bridging through ten, modelled on a 100 square, can also be shown on a number line and can help children become more efficient:



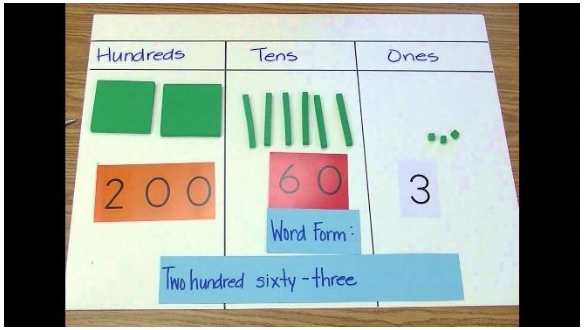
PROGRESSION THROUGH ADDITION: Models, Images & Written Methods

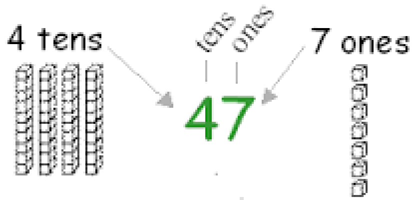
- Children should continue using empty number lines and hundred squares with larger numbers, bridging 100 and re-ordering the calculation when appropriate. Use this image when adding time, other measures, and money:

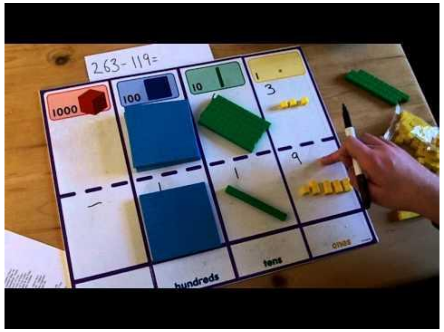
38 + 86 = 124



- Understanding the value of the digits in two then three digit numbers, can be supported by '**building the number' with Dienes** apparatus. Dienes are a great representative tool because they show the size of each digit, unlike coins or place value counters which might be the same size but different values. Use **Dienes** with the children, placing them in place value charts labelled 100s, 10s and 1s (not units). Remember to give the children **pictorial representations of the Dienes alongside digits** on any printed activities. This pictorial step must not be rushed over!

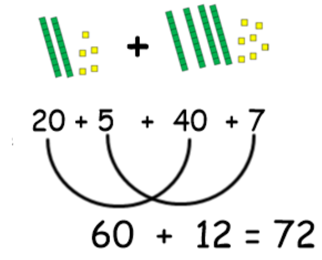

Hundreds
Tens
Ones
200
60
3
Word Form:
Two hundred sixty-three


4 tens
tens
ones
47
7 ones


263-119=
1000
100
10

PROGRESSION THROUGH ADDITION: Models, Images & Written Methods

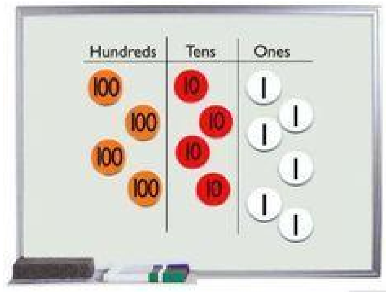
- Using Dienes supports partitioning and adding. This is a pictorial and abstract representation of what should become a mental strategy as children move through the school. This example shows 25 + 47 regrouped to form 60 + 12:



- Children will begin to partition two digit numbers horizontally, and then to add tens and units vertically.
  This is called **expanded addition** and can be recorded like this:

37 + 22 =
30 + 7
20 + 2
50 + 9 = 59

This stage must be supported by using **Dienes apparatus and pictorial** representations as shown above… lots! When the children are ready to understand that objects with the same size can represent different values, they can use **place value counters** to support expanded addition vertically.

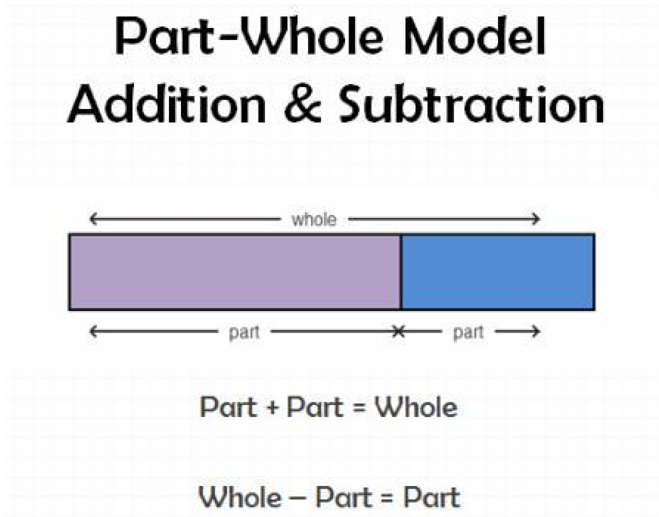


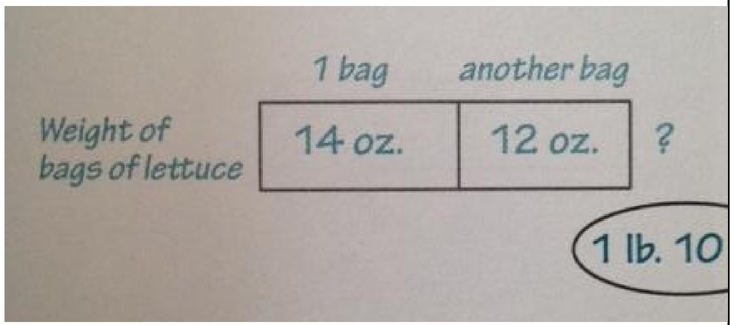
- Draw the children's attention to the fact that when adding mentally, we start with the most significant digit, but when adding using column addition, we start with the least significant digit.

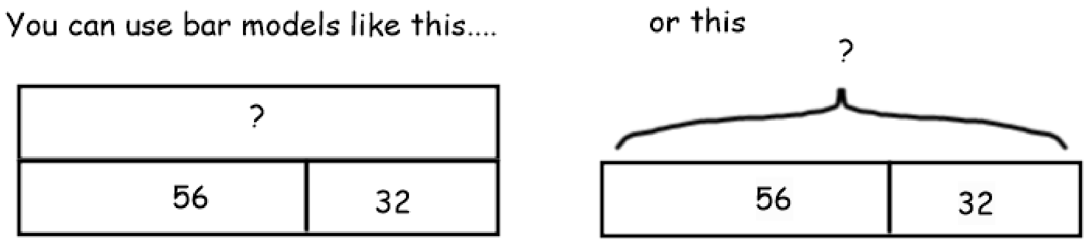
Note: The children should be encouraged to add two digit numbers mentally as soon and as much as they are able, initially with no' carrying' involved (ie. ones totalling no more than 9). They should then work towards adding larger numbers with place value counters, working on carrying over and exchanging 1s for 10s when there are more than 9 in the ones column, for example. If they can do a sum mentally efficiently, they should do so and throughout the school, mental maths should be encouraged if the numbers lend themselves. ***This means you must choose your digits with the utmost care so the method you are teaching is necessary!***

PROGRESSION THROUGH ADDITION: Models, Images & Written Methods

Children must begin to recognise and use the inverse relationship between addition and subtraction early on. When solving real life problems, bar modelling is a key technique for the children to pictorially represent their thinking. If done to solve addition problems, it will help children with subtracting to 'find the difference' (see subtraction) because it shows the whole – part relationship:
This model is only appropriate for children who understand that a bar can represent a number bigger than 1 and for children who have developed or are developing the strategies shown above (using a number line to add or partitioning):







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<tr><th colspan="2">PROGRESSION THROUGH ADDITION</th></tr>
<tr><th>Mental Maths Supporting this Stage</th><th>Models, Images & Written Methods</th></tr>
<tr><td>
Factual Fluency
● Continue to rehearse and revisit addition and subtraction facts for all numbers to 20 and 100. For pairs to 100, start with multiples of 10, then 5, then any number).
● Give 10 or 100 more or less than a given number (spider counting and using knowledge of place value).
● Count on and back in hundreds, tens and ones.
● Round numbers to the nearest 10 or 100
● Recall of pairs that make 1 whole or £1 or 1m.
Mental Strategies
● Use knowledge of place value to a one digit number, a multiple of 10 or a multiple of 100 to any 3 digit number.
● Use knowledge of place value: If I know… I also know…
e.g. 140 + 150 is related to 14 + 15
● Partition numbers in different ways e.g. 146 = 100 + 40 + 6 and 146 = 130 + 16 (this type of work is crucial for columnar addition).
● Add using near doubles and nearly numbers.
● Add more than 2 numbers (piles o’ numbers!) by finding pairs to 10 or doubles in the sum.
● Add two, three or more coins by regrouping coins of the same denomination, bridging £1.
● Use the above strategies to add decimal numbers e.g. 1.1 + 1.2 is a near double!
</td><td>
● Use number lines as a key strategy for rounding to the nearest 10 or 100. If they can place it on a number line, they can round it by finding the café numbers (multiples of 10 or 100) which it is between.
● Children must continue to use empty number lines to jot down notes for mental addition, for example, for adding nearly numbers then adjusting. All work done using number lines is brilliant for place value and securing understanding of numbers’ relative sizes.
● At EP, Year 3 is the year when efficient columnar addition is secured for most children. They will continue to use expanded horizontal methods to add three digit numbers and exchanging ones for a ten should be begun now.
1) Addition must be modelled using dienes first with parallel teaching showing expanded notation to record this.
263 + 128 =
Do this…


alongside this…
200 + 60 + 3
100 + 20 + 8
300 + 80 + 11 = 391

Note: Only carry over from the ones first, then with ones and tens. These micro-steps matter! Watch out for when a column adds to 10, or 100; sometimes this throws the children because it is represented by a zero. Show how the 11 becomes a ten and a 1, and that this can be put into the tens column now.
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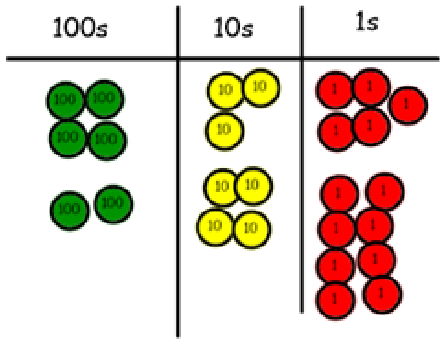
PROGRESSION THROUGH ADDITION: Models, Images & Written Methods

2) Move on to modelling with place value counters. Make sure the children have large place value grids with enough space to explore this method. In their books, the additions must be recorded clearly with enough space. Be consistent in how you lay these out. It is important for accuracy

3) Most children will move on rapidly, with understanding, to compact columnar addition, if you show expanded addition alongside compact addition. When you move compact addition, the place value becomes implicit instead of explicit and you will lose some children! Make sure you keep doing this ‘split screen’ teaching to show expanded and compact for the same calculations:

435 + 248 =



100s	10s	1s
400	30	5
200	40	8
600	80	3
	10	

to

100s	10s	1s	
4	3	5	
2	4	8	+
6	8	3	
1			

You may choose to cross out the carried over digit when it's been added on.

Note: Some children might find this intermediate step between expanded addition and compact useful:

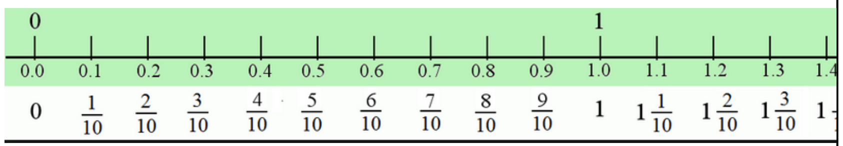
```
  4 3 5
  2 4 8 +
    1 3   (5 + 8)
    7 0   (30 + 40)
  6 0 0   (400 + 200)
  6 8 3
```

PROGRESSION THROUGH ADDITION: Models, Images & Written Methods

- When the children have mastered place value of whole numbers, they must focus on decimals (tenths and hundredths) so using models images to ensure they understand this place value, as well as adding money and measures are important. Ensure that lots of work is done placing decimal numbers on number lines (decimal ITP – interactive teaching programme online - is good for showing this) and if children are not secure, they must explore the meaning of tenths and hundredths with these three elements before they add:

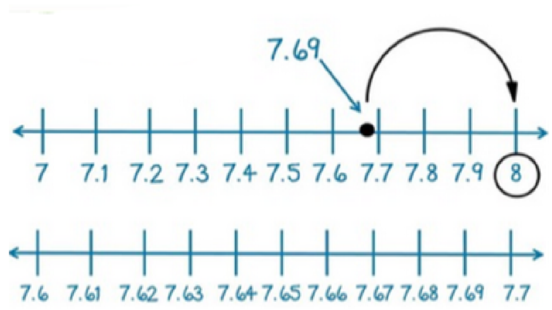
Tenths and hundredths as fractions

When teaching fractions **as** numbers, count in steps of 0.1 and ensure children understand 1/10 = 0.1 and 10/10 = 1 then do the same for 0.01 and 1/100. Take time and care when you go from 0.9 to 1 then 1.1, avoiding the children saying 0.9. 0.10

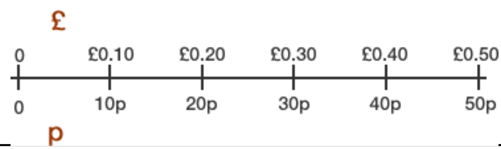


Decimals on a number line

Place decimals on a number line between whole numbers and work on avoiding good mistakes with place value.



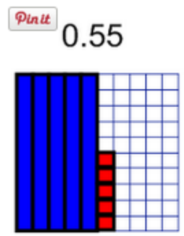
The decimal ITP (interactive teaching program) which can be found easily online, is good for showing these 'spaces in between'. Linking this to a ruler and exploring m, cm and mm, and to money is a good idea at this stage.



A money 100 square might also be useful.

Dienes as decimals!

Use Dienes with the 100 slab representing 1, a rod representing 0.1 and a cube representing 0.01. Moving from the number line to this model is a bit like the jump children make in Year 2 from the number line to using a number square and partitioning. Remember this is a big step!
Build numbers using this resource, and pictures of this resource:

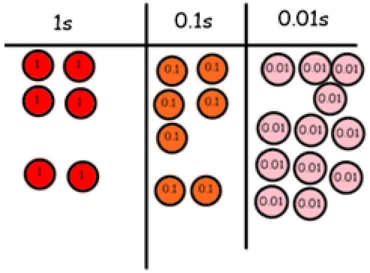


PROGRESSION THROUGH ADDITION: Models, Images & Written Methods

- Ensure the stages above are gone through when first adding tenths (or 10p and 1p). Use 0.1 and 0.01 place value counters. I was careful in this example that no column added up to 11 because this might cause confusion initially.

Split Screen Teaching!

4.54 + 2.28 =



1s	0.1s	0.01s
4	0.5	0.04
2	0.2	0.08
6	0.8	0.02
	0.1	

The carried over 0.1 is shown below the line here.

1s	0.1s	0.01s	
4.	5	4	
2.	2	8	+
6.	8	2	
	1		

You may choose to cross out the carried over digit when it's been added on.

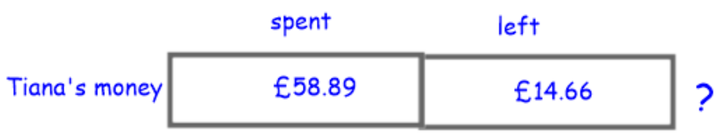
The progression through doing these types of calculations should be as above, carrying once, then twice etc. with these tweaks:

- add one place decimal numbers
- add two place decimal numbers
- add two amounts with different numbers of decimal places.
- add amounts with no ones, just tenths and hundredths.
- add piles o' numbers (three or more amounts)

Remember to use split screen teaching to show compact methods so that children can see a model which makes place value clear and explicit alongside the most efficient method where place value is implicit.

Note: think through examples for written calculation carefully with no near doubles etc. which would mean a mental method would be quicker.

- Bar modelling is a key image for representing problems involving real life time, measures and money. Children must be confident in using strategies above to actually calculate answers:



Tiana spent £58.89 on books. She had £14.66 left. How much money did she have at first?

<table>
<tr><th colspan="2">PROGRESSION THROUGH ADDITION</th></tr>
<tr><th>Mental Maths Supporting this Stage</th><th>Models, Images & Written Methods</th></tr>
<tr><td>

Factual Fluency

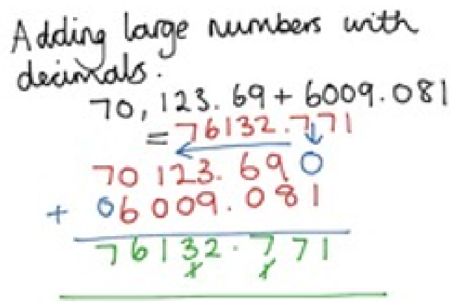
- Know what must be added to any four digit number to make the next multiple of 1,000 e.g. 4,087 + ? = 5,000
- Know what must be added to a one, two then three place decimal to make the next whole number e.g.
  7.2 + ? = 8   7.21 + ? = 8   7.219 + ? = 8

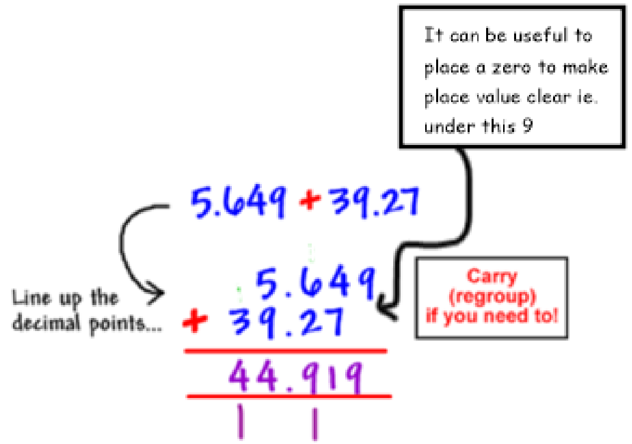
Mental Strategies

- Use previously learnt strategies with larger and decimal numbers ie.
  - doubles and near doubles 2.1 + 2.1
  - nearly numbers and adjusting 4.5 + 1.9.
  - partitioning and recombining 421 + 512
- Partition numbers up to a million.
- Count on or back in minutes and hours, bridging through 60 minutes.

</td><td>

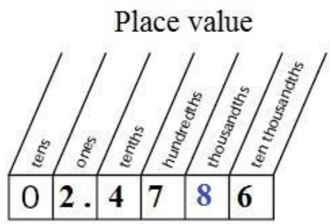
Continue to use vertical columnar addition efficiently and accurately, ensuring that each calculation could not be done quickly mentally (ie. is it a near double etc.?) and that decimal numbers are lined up correctly. Children might want to put zeros in to ensure their place value is

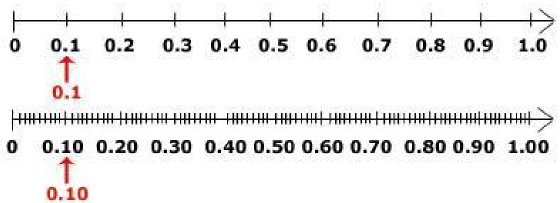




correct:

- Explore three place decimals in Year 5, going through the stages laid out in Lower KS2. Place value counters, number lines and expanded addition can be used to ensure understanding.





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PROGRESSION THROUGH SUBTRACTION

Mental Maths Supporting this Stage

***See addition** for first steps (ie. knowing number names). The children must first understand the relationship between addition and subtraction.*

- Use 'more' and 'fewer' to compare sets of objects.
- Count back from 20 to 0 in familiar and practical contexts.
- Find one less than a number by counting back.

Factual Fluency

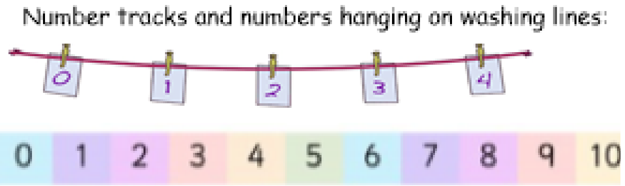
- Know the 'story of' numbers up to 5 then 10… number bonds!
- Move on to the 'story of' all numbers to 20, using these to solve subtraction calculations quickly e.g. 7: 3 + 4, 5 + 2, 6 + 1, 7 + 0, 7 – 6 = 1, 7 – 5 = 2 etc.
- Develop fast subtraction of any single digit number from a multiple of 10.
- Quickly recall halves for all even numbers to 20 e.g. half of 18.

Mental Strategies

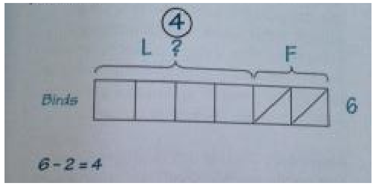
- Relate subtraction to finding how many are left when some are removed.
- Relate subtraction to 'finding the difference' in practical contexts
- Subtract any number from another below 20.

Models, Images & Written Methods

Ensure you are counting back on tracks and lines just as much as you are counting on…



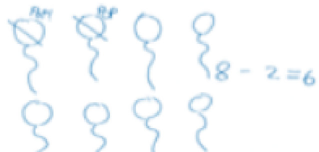
- Use up to 10 then 20 objects to model 'taking away'.
- Make comparisons and finding the difference in practical contexts.

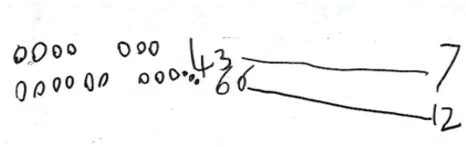


- As with addition, bar models where one square represents one object can be used to represent problems such as this one where two birds flew away, once the children understand that symbols can represent objects.

- There are many classic sesame street videos which encourage counting, comparing and subitising… explore! You could start with Bubble Gum Math

See addition for principles of children's early recording. The – and = should be used to record calculations by the end of Reception and all types of mathematical recording of calculations should be encouraged and OWLed!





PROGRESSION THROUGH SUBTRACTION

Mental Maths Supporting this Stage

Factual Fluency

***See addition** for number pairs and 'story of' where addition and subtraction are linked.*

- Given a number, identify one less
- Count back across 100, or from any given number. Spend as much time counting backwards as forwards! Use wormy ones!
- Count back in ones, twos and tens.
- Know subtraction facts for all numbers to 10 then 20 then 100.
- Knowing how far the next café number (multiple of ten) is (this is essential for finding the difference).

Mental Strategies

- Takeaway numbers by using 'story of' e.g. 18-3 = 15 because 8-3 = 5
- Use patterns of similar calculations e.g 20 -0 = 20 20 – 1 = 19 20 – 2 = 18
- Subtract 10s and multiples of 10 from numbers using a 100 square and spider counting.
- Subtract nearly numbers by subtracting ten then adjusting e.g. - 9 by subtracting 10 (spider) then adding 1.
- Count back in repeated steps of 1, 10, 100 or 1,000 e.g. 86 – 52 = 34 (by counting back in tens and then ones)
- Find a small difference by counting up e.g. 82 – 79 = 3

Models, Images & Written Methods

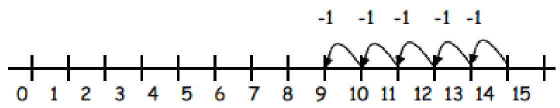
Subtraction should be explored in two ways: taking away, and finding the difference. The notes below show both progression through take away and then difference. Both should be taught in Year 1 and Year 2.

Be clear on whether you are teaching taking away, which we call **robber** subtraction at EP, or finding the difference which we call '**frog finds the difference'.**

TAKE AWAY

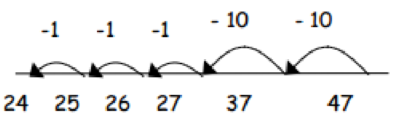
- Use number lines to take away by counting back in wormy ones. This is 'robbing' because we are taking away an easy amount that's quick to snatch!

13 - 5 = 8

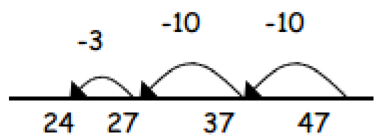


- Move on to taking away tens then ones on a number line:

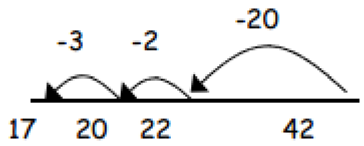
47 - 23 = 24



then grouping the 1s in one jump



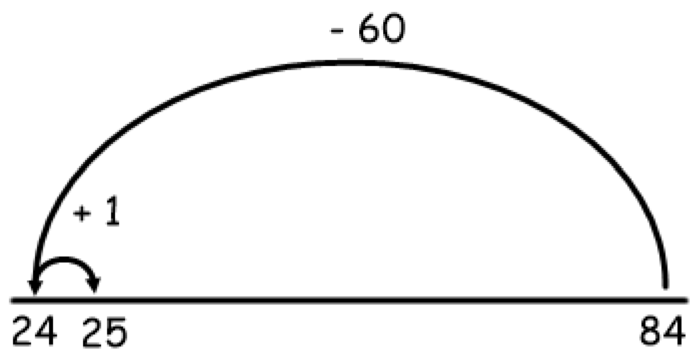
then grouping the tens in one jump…



For a ‘take away’ such as 47 – 25, partitioning the ones to bridge a café number might help.

- Counting back in tens then adjusting can be done on a number line and a 100 square:

84 - 59 = 25



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25		27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Model counting back in **wormy ones** on the number line then on the 100 square. Then count back (using the spider) in 10s. We call this **spider counting**.
- Once this is mastered, subtract nearly numbers e.g. - 11 by taking away 10 then 1. The next step would be to subtract 9 by taking 10 then *adding on 1… can they see why?*

- Explore the fact that subtraction is not commutative but avoid saying the larger number *has* to go first.
- Children should record and complete equations, understanding the = symbol as showing equivalence rather than ‘is the answer’. They use number bonds and the inverse relationship to record ‘fact families’. e.g. 3 + ∆ = 10

Try some other equations which require - balancing: 10 - 2 = 4 + ∆

<table>
<tr><th colspan="2">PROGRESSION THROUGH SUBTRACTION</th></tr>
<tr><th>Mental Maths Supporting ‘Take Away’</th><th>Mental Maths Supporting ‘Difference’</th></tr>
<tr>
<td>Factual Fluency
● Continue to revisit subtraction (and addition) facts for all numbers to 10, 20 and 100. (link to money £ and p)
● Give 10 or 100 less than a given number.
● Subtract ones, tens, or hundreds from any 3 digit number.
● Count back in thousands, 100s, 50s, 25s, 10s, tenths and hundredths fluently. Be imaginative! Count back in 0.5s from 20.4 for example!

Mental Strategies
● Subtract a two digit or three digit number from any three or four digit number when no exchanges are needed.
● Subtract a near multiple of 10 (nearly number) and adjusting e.g. 453 – 29 by -30 then adding 1.
● Subtract any decimal number that is nearly a whole number, and adjust.
● Count back in hours and minutes, bridging through 60.
● Partition numbers in different ways e.g

146 = 100 + 40 + 6 or 100 + 30 + 16 (this is crucial for columnar subtraction).
● Subtract increasingly large numbers e.g. 12,462 – 2,300 using place value without exchanges.</td>
<td>Factual Fluency
● Continue to revisit subtraction (and addition) facts for all numbers to 10, 20, 100 then to 1 for decimal numbers.
● Recall quickly all pairs that total 100 (multiples of 5 then any number), then pairs to 1,000.
● Round to the nearest 10 or 100
● Know how many to the nearest 10, 100 or 1,000 (crucial for finding the difference).
● Know difference between decimal numbers and the next whole number e.g. 7.21 + ? = 8
● Round decimal fractions to the nearest hundredths and thousandths.

Mental Strategies
● Adding mentally by partitioning and adding the most significant ‘jumps’ on our number line first’.
●</td>
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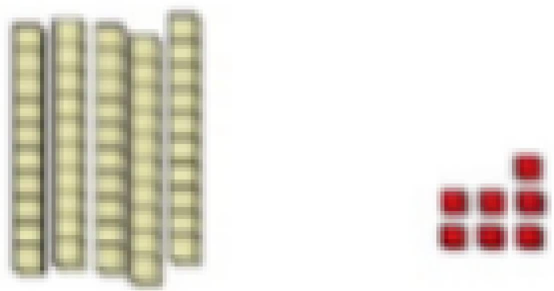
PROGRESSION THROUGH SUBTRACTION: Models, Images & Written Methods Continued

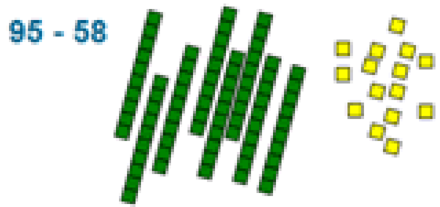
TAKE AWAY CONTINUED

- Children who have mastered taking away on a number line in tens and ones, and who are secure with place value, adding by partitioning and using dienes, can begin to explore subtraction by partitioning dienes. This is the first step towards columnar subtraction which is the focus for most children in Lower KS2. This needs to happen without exchange first!

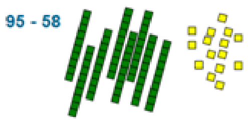
  57 – 32 could be done without exchange using these:

  However, this would requiring exchanging a ten for ten ones:

When columnar addition has been securely taught (usually in Year 3) expanded then compact subtraction can be taught, making Year 4 a crucial year for subtraction for most children.



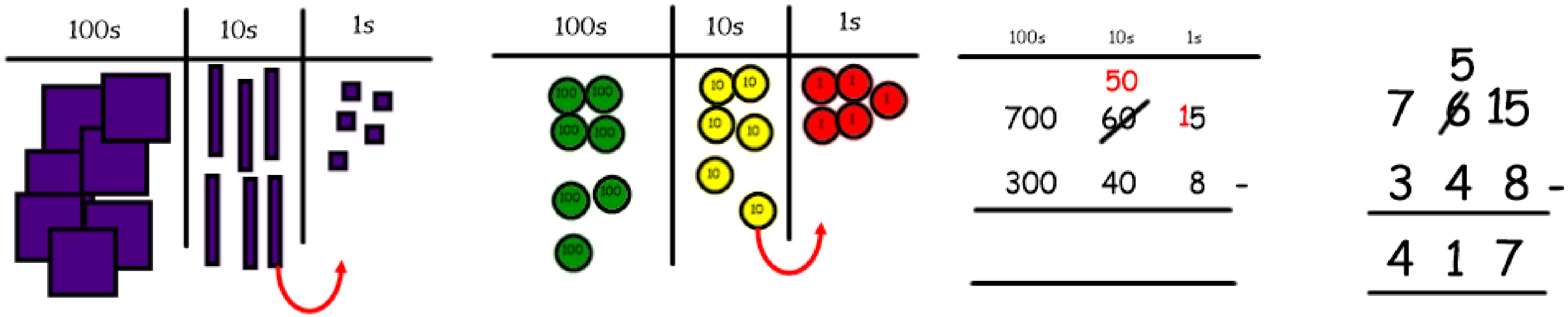
- First, explore with the children how we can take away using partitioning and dienes where there is no exchange. Place value counters can be used alongside this. NB. You only need to make the subtrahend (first number) in the calculation with your apparatus!

PROGRESSION THROUGH SUBTRACTION: Models, Images & Written Methods

TAKE AWAY CONTINUED

- Next show a subtraction recorded in expanded form, while you model this with Dienes apparatus, physically, exchanging a 10 for 1s and crossing out. Give yourself lots of space to model this!

Do some split screen teaching, showing what happens when we 'can't do it' and need to exchange.



- Start with the ones. Oh no! 5-8! You can't do it (unless we go into negative numbers!)
- Let's exchange a 10 with the 'banker' for 10 ones.
- Now turn around, and start again! Now we have 15 ones and we can subtract 8.

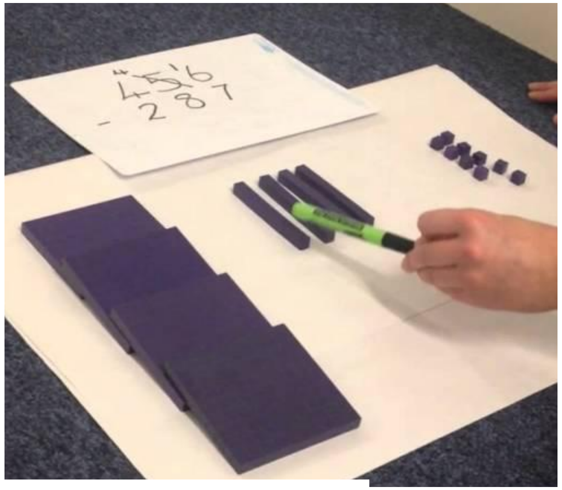
- When the children have mastered this, show a split screen between the expanded method and compact. Exchanging should now make sense! Sometimes children might partially use the place value counters, then complete the calculation using a written method when they have *seen* how it works. This is fine and to be encouraged! Each time you move towards compact, you lose some children because place value becomes implicit so make sure you keep modelling split screen when you progress e.g. when there's a zero in the 10s.

PROGRESSION THROUGH SUBTRACTION: Models, Images & Written Methods Continued

Note: Whenever you move to a method where the place value is implicit – compact columnar subtraction – some children get lost! This means that you must keep doing your split screen teaching and ensuring understanding is there.

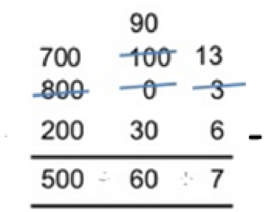
- This is the progression in columnar subtraction that should be taught and practised conceptually and thoroughly:
  - Subtraction of three or four digit numbers with an exchange needed for the ones.
  - Subtraction of three or four digit numbers with an exchange needed for the tens or hundreds

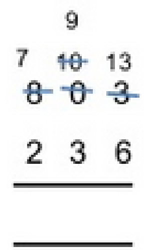
Top tip: ensure no digits in the subtrahend become 0 when exchanging takes place at this stage. Watch out for the classic 'good mistake' of 2 – 8 = 6!

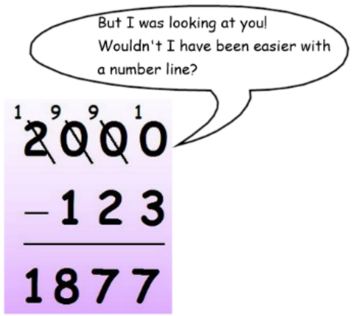


- Several exchanges.
- What happens when there is a zero in the tens column and you need to exchange? Model going to the hundreds column, exchanging this for ten tens, then exchanging one of these for a ten ones. This requires lots of modelling and it's very clear to the children why it needs to happen when done with dienes!

This becomes…







- What happens when there is a one in a column where you need to exchange? This leaves zero and then requires the previous step.
- What happens with a calculation like this: 35.2 – 16.35? Teach children to line up columns accurately, writing in the zero!

Continue to point out how many errors *can* occur with this method if it's 'looking at you'. We say 'looking at you' because we can imagine the zeros are eyes! Many zeros make decomposition difficult although the method will work when done correctly): 2,004 - 1,897 =

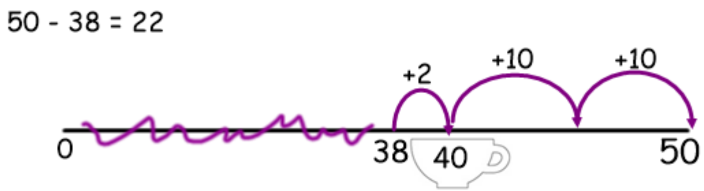
PROGRESSION THROUGH SUBTRACTION: Models, Images & Written Methods

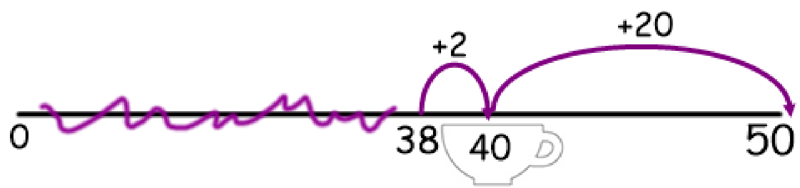
DIFFERENCE: Minuend - Subtrahend = *Difference!*

- Explore the **difference** between two numbers that are close together throughout Year 1 and Year 2 starting with comparisons of two amounts side by side:

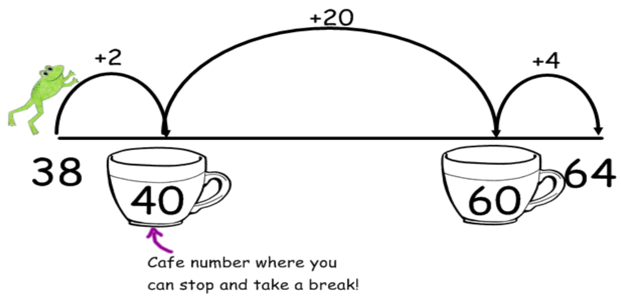
This model contains the whole 'fact family' of 3 + 5 = 8 and can be represented with Cuisenaire rods and as a bar model as above. Note that comparisons can be made horizontally or vertically. Children often understand the 'difference' between two heights easily!

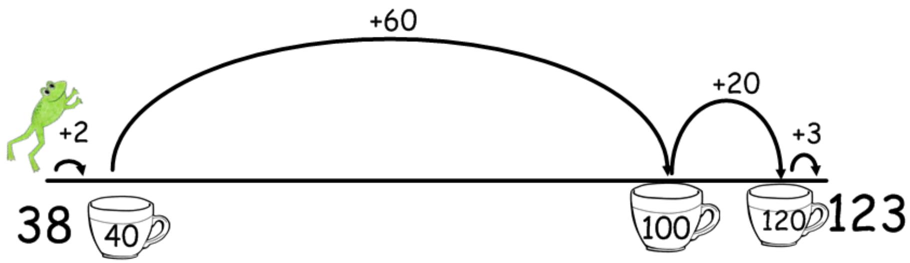
- When finding the difference, at first always draw your number line from 0 and 'subtract' the bit you are subtracting by scribbling it out. Now you have to find the difference or 'how much is left'. At EP, we say we *could* count in 1s but this would take ages so we count to the next café number and have a slurp of our drink (whole class mimes taking a drink)! This gives us a rest. Now we hop to the next café, writing down how far we've gone above each jump. We might be able to jump more than one ten at a time:





- Next, children will be able to start from the smaller number and stop writing zero! Model counting up to the café numbers A LOT and remember to add these jumps to find your 'difference'. You can then start to bridge 100. Remember you can check with the inverse! These examples show 64 – 38 = 26 and 123 – 38 = 85



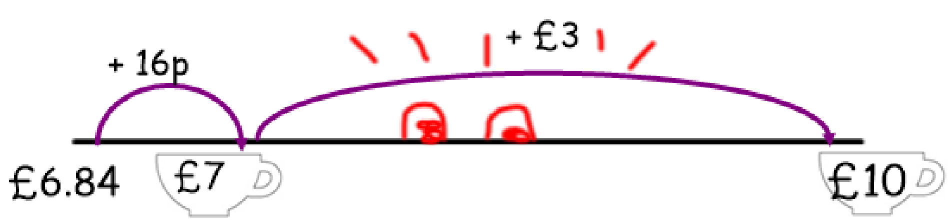


PROGRESSION THROUGH SUBTRACTION: Models, Images & Written Methods

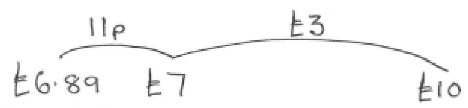
DIFFERENCE: Minuend - Subtrahend = *Difference!*

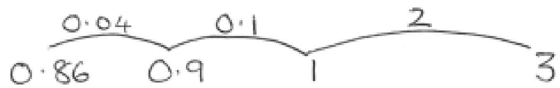
- Finding the difference using a number line should be taught, developed and used in these instances throughout KS2, moving through the progression outlined above at the right pace for the children. NB it's not 'cleverer' to do column!
    - When the numbers are close together, whether it be 4 digit numbers or decimals with 3 places.
    - When the numbers are looking at you! e.g. 2,001 – 1,856 draw little pupils in the zeros to make them look like eyes!
    - When finding time difference
    - When subtracting from a café number including money (finding change) and measures e.g. change from £5 or subtracting from 1l or 1km. (this is really because it's looking at you! £5.00 and 1,000ml)

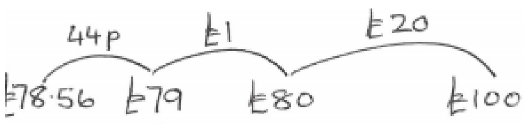
£10 - £6.84 = £3.16

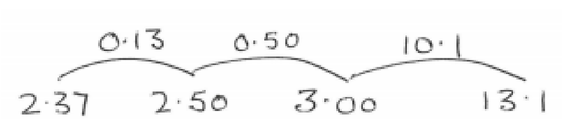


- When they are very secure with finding the difference, they should still be encouraged to record informal jottings to help support their mental maths, based on the models used lower down the school:









PROGRESSION THROUGH SUBTRACTION: Models, Images & Written Methods
When upper junior children are *totally* fluent, accurate and *very* secure with formal written subtraction, you could show them some alternative methods to develop their reasoning and for mathematical interest, asking how these methods work:

A 'sandwich sum' version of the empty number line where steps up the number line from the bottom number to the top are recorded:

```
   71
 - 46
    4 (46-50)
   20 (50-70)  +
    1 (70-71)
  25
```

The negative number method ("you <u>can</u> take 6 away from 1 - it's minus 5")

```
   71
 - 46
   -5  (1-6)
   30  (70-40)
   25  (30-5)
```

```
754
286
  -2   (4-6)
 -30  (50-80)
 500  (700-200)
468
```

<table>
<tr><th colspan="2">PROGRESSION THROUGH MULTIPLICATION</th></tr>
<tr><th>Mental Maths Supporting this Stage</th><th>Models, Images & Written Methods</th></tr>
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Factual fluency

- Encourage talk about counting the same number again and again e.g. while counting pairs of wellies… “There’s two, and two, and two again.” or counting fingers… “There’s 5 and 5 again.”
- Count in 2s, 5s and 10s. Don’t stop after 20 or 50 or 100! Count back as well!
- Learn and recall doubles and halves of all numbers to 10.
- Know odd and even numbers. (Numicon!)
- Learn doubles of all numbers to 20 and corresponding halves.

Mental Strategies

- Notice and talk about patterns when counting in groups, looking at the last digit.
- Count in 5s and 10s using clocks and fingers.
- Double any multiple of 10 up to 50 then any multiple of 5 up to 50 by using related facts and partitioning
- Find the total number of objects when they are organised into groups of 2, 3, 5 or 10 by counting in groups.

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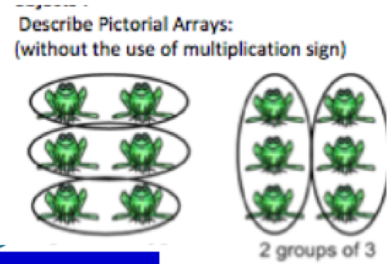
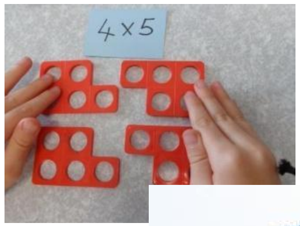
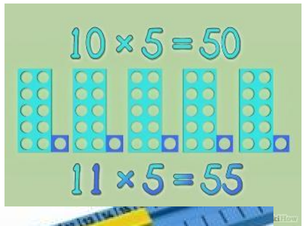
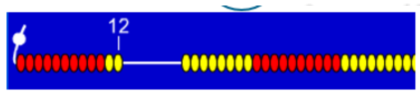
Multiplicative thinking is when children can count in ‘groups of’ or ‘units of’ other than 1. This is a huge step in development, occurring for most children by the end of KS1. Therefore, thinking in ‘groups of’ supported with practical resources and built upon progressively.

- Children should use resources and pictures to solve multiplication problems such as ‘How many wheels on 3 cars?’ If there are 2 windows on each house and 4 houses, how many windows are there?’
- Count in multiples of the same number using Numicon, Cuisenaire rods, objects or beads on a string:









- These models all show multiplication as **repeated addition.**

This that’s

- Develop an understanding of multiplication as **scaling** e.g. piece of ribbon is 5cm. How long is the ribbon 4 times as long?

- Introduce the X symbol as multiplied by, AKA ‘groups of’.

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</table>

PROGRESSION THROUGH MULTIPLICATION: TIMES TABLES

Multiplication facts should be memorised (see [Mastery of the Stick]() below) and chanted every day from Y2 onwards ('counting in' starts in Year 1), either as part of the mental oral starter or other times as appropriate within the day. Times table weekly tests start in Year 3 and continue into Year 6 where the emphasis is on speed or recall and application to other facts e.g. 0.4 x 0.03. Teachers should ask questions about related facts that supersize, super shrink or are about division e.g.

What are 6 sevens? What is 60 X 7? What is 0.6 X 7? What is 42 ÷ 6? What about 43 ÷ 6?

Teachers should also reinforce the meaning of these tables in lessons, using images such as Numicon and arrays. Use empty boxes and symbols throughout to explore inverse relationships:

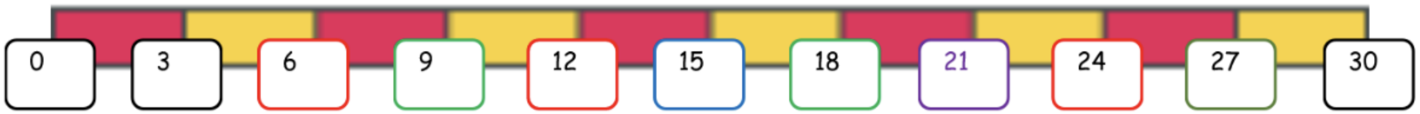
□ x 5 = 20   3 x △ = 18   □ x ○ = 32

TEACH TIMES TABLES IN THIS ORDER:	DIVISIBILITY TESTS
2 & 10 X tables (NC: Y1 & Y2) These two are essential because of doubling and place value.	Multiples of 2 are even. Multiples of 10 end with 0.
5 X table (NC: Y1 & Y2) This has a great pattern to explore and it's half the 10s	Multiples of 5 end with 5 or 0.
3 X (NC: Y2)	The digits of multiples of 3 add to 3, 6 or 9.
Review 2 X table then learn 4 X table and the 8 X table (NC: Y3) Double and double to get 4s; double, double and double to get 8s!	The last two digits of multiples of 4 are multiples of 4 because 100 is a multiple of 4 e.g. 2**48** is a multiple of 4 because 48 is. If you halve a number and it passes the test for multiples of 4, it's a multiple of 8!
Review the 3 X table and learn 6 X tables (NC: Y4)	Multiples of 6 are even multiples of 3.
9 X and 11 X tables (NC: Y4) More patterns to spot so these are easy wins!	 The digits of multiples of 9 add up to 9. Finger trick! Up to 9 X 11, the pattern is clear. After that, you can work them out using 10 X then add 1X the factor.
7 X table (NC: Y4)	No nice test or trick so memorise!
Discuss how we know the 12 X table if we know the rest!	Multiples of 12 pass the tests for 3X and 4X.

Mastery of the stick!

*This is based on video found at: **www.youtube.com/watch?v=yXdHGBfoqfw** or google 17 X table Jill Manssergh*

1) Spend several lessons memorising multiples using the 17X table process (as below) This would work for 'counting in' fractional/ 25s etc steps too!
2) Spend lots of time chanting a la EP e.g '3 3s are 9'..., going backwards too, random questions and extend the questioning and stick to 'if we know x what else do we know'
These are definitely two different phases and I wonder if we are doing enough on phase 1!
Counting stick for 3 x table (as example):



- Place 0, 3 and 30 and say we are learning our 3 X table… the multiples of 3!
- Double 3 to place 6
- Double the 6 to place 12 (good mistake of putting it in the 3rd multiple spot avoided)
- Double the 12 to place the 24 (the 8th multiple)
- How do we find this one? (9... the 3rd multiple) we can add 3 to 6 or take 3 away from 12… she called the 3rd multiple the key!
- Double the key to place the 18
- How do we find the middle… give me 3 ways! (half of 30, add 3 to 12 or take 3 away from 18) Put finger on the nose for the 'middle' and always hold the counting stick in the middle.
- How do we find the 9th multiple… give me 3 ways (take 3 from 30, add 3 to 24 or 3 x the key)
- How find the 7th multiple… I can't remember it so it's your job!
- Now pause at this point to reassure class that if you make a mistake, no-one will notice because we're all focused on the 3 x table! Say you are a great teacher and they are a great class!
- CHANT Remove 0, 3 and 30 …. Repeat lots and remind 'what's the key?.. double it?' etc. jumping about on the stick.
- Re-CHANT remove 6, 12 and 24 after saying 'double it is...' and 'double it is...'
- Re-CHANT removing the middle. Keep revising ones you've taken off... 'what times table are we learning? And what is 10 X...?'
- Re-CHANT removing the key and its doubles. Revise all the ones that have gone.
- Re-CHANT removing the 'one you can't remember'… 21 Revise all
- Re-CHANT on the naked stick and give yourself a clap!

PROGRESSION THROUGH MULTIPLICATION

Mental Maths Supporting this Stage

Factual Fluency

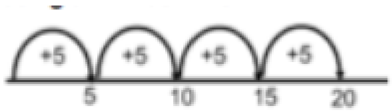
- Learn multiplication facts for the 2, 5 and 10 X tables, and corresponding division facts.

Mental Strategies

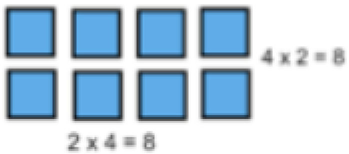
- Partition to double numbers by doubling tens and ones separately, then recombining.
- Use knowledge that doubling is equivalent to x2
- Use knowledge of multiplication facts to find totals e.g. 3 groups of 5 = 15 objects. (arrays)
- Multiply by 10 by shifting the digits to the left, and explore whether this always works. This lays the ground work for long multiplication.

Models, Images & Written Methods

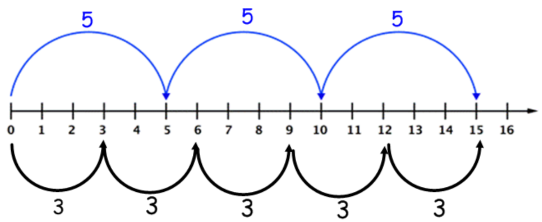
- Make explicit links between multiplication and repeated addition by counting on in groups on a number line:



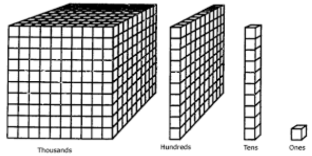
- Continue to work on understanding multiplication as grouping using arrays. This also helps with understanding the law of commutativity and linking multiplication to division.



Children should know that 3 X 5 has the same product as 5 X 3. This can be shown on a number line:



- Use a slider to show how digits slide left as we multiply. Use 'ones' rather than 'units'. Remember to reinforce this by using dienes to get the sense of 'scaling up'. Choose manageable examples using dienes!



PROGRESSION THROUGH MULTIPLICATION

Mental Maths Supporting this Stage

Factual fluency

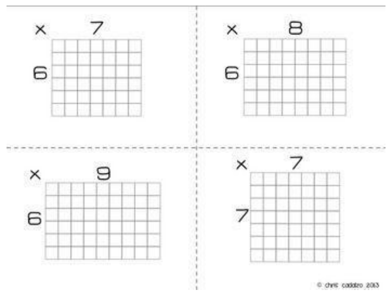
- Count in steps of 2, 3, 4, 5, 8, 10, 50 and 100 from 0.
- Learn and recall the 9 and 11 X tables.
- Learn and recall multiplication and division facts for the 4 and 8 X table.
- Learn and recall the 3 and 6 X table.
- Continue to build speed and fluency with doubling whole numbers to 50.

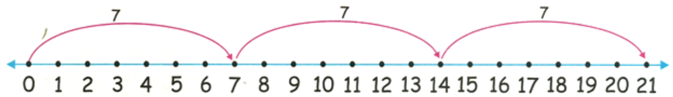
Mental Strategies

- Multiply whole numbers by 10, 100 and then 1000 (essential for grid method).
- Find multiples of 8 by doubling multiples of 4
- Find multiples of 6 by doubling multiples of 3.
- Find multiples of 9 by multiplying by 10 and subtracting one group.
- Find multiples of 11 by multiplying by 10 and adding one more group.
- Use doubles of smaller numbers to derive doubles of multiples of 50 to 500 e.g. double 150 by doubling 15.

Models, Images & Written Methods

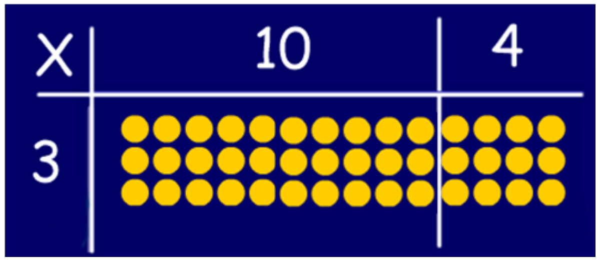
- Keep using number lines alongside arrays when teaching new times tables:





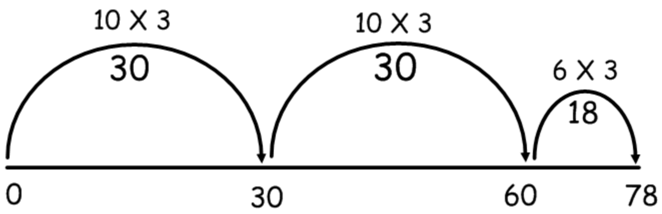
- Begin to link arrays with grid method to multiply a one digit number by a two digit 'teens' number. Multiplying by 14 is a good place to start, since you can explore 10 X the number and 4 X the number pictorially as an array quite manageably:

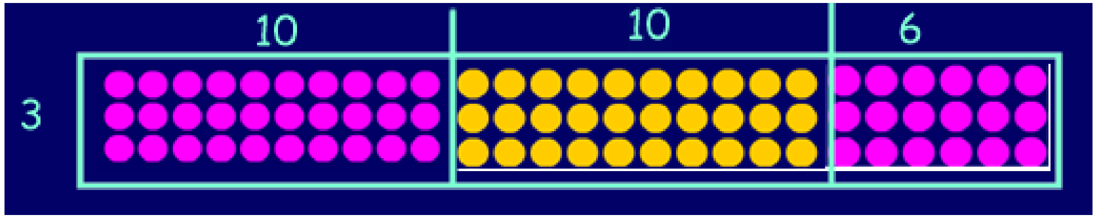
This model is best **<u>not</u>** used when multiplying by 4 (double and double mentally) or 5 (multiply by 10 and half). It is effective when you stick to the same X table that you're rehearsing and learning, so this image would work well alongside learning the 3 X table and all the calculations given would involve X 3.



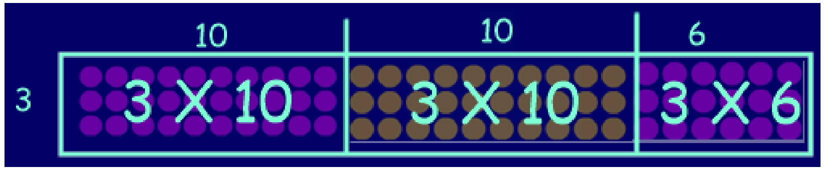
PROGRESSION THROUGH MULTIPLICATION: Models, Images & Written Methods

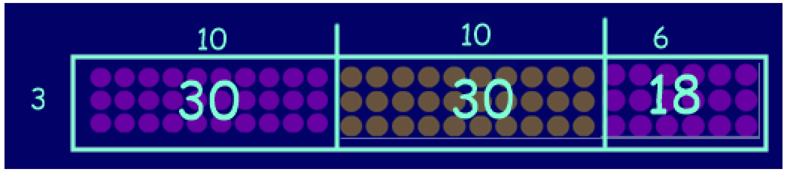
- Use a number line to model how the distributive law of multiplication works. Do some split screen teaching, showing the same multiplication modelled using an array:
  Here, 26 X 3 = (10 X 3) + (10 X 3) + (6 X 3)



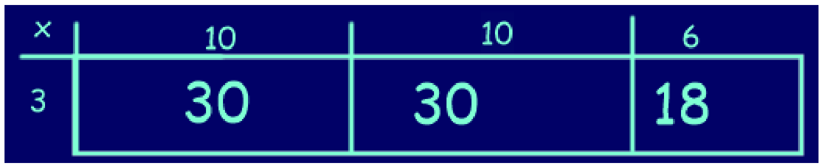


- Move on to an open array, phasing out the need for the counters/squares within the array:





- You can now progress to drawing the grid and finding the 'area' of the rectangles, then when children can use place value to multiply 3 X 20, combine the 10s:



Ensure the children understand *why* we are adding the area of the grid, showing the array models again as needed.
Children who are not yet fluent with times tables must use a times table square to support learning the method… it's good practice!

PROGRESSION THROUGH MULTIPLICATION: Models, Images & Written Methods

SHORT MULTIPLICATION is when you multiply by a one digit number:

- It is very important that children see the link between knowing 3 X 2 and 3 X 20 because this is why we bother to learn times tables!
  Teach the children the “how many zeros in the question… that’s how many in the answer” trick.
  e.g. to solve 40 x 3 use 4 x 3 = 12 adjusted with one zero = 120
  to solve 500 x 300 use 5 x 3 = 15 adjusted with 4 zeros = 150 000
  NB Careful when 5x4 = 20 or 8x5 = 40 : for 50 X 4 you need to adjust with one zero in addition to the one after 20… so the answer is 200! the 0 in 20/40 doesn’t count! Avoid saying “add zero!” and keep reinforcing that what is actually happening is the digits shifting to the left.
- ‘Split screen teaching’ is needed to move from grid method to the expanded then the compact columnar method:

x	20	6
3	60	18

```
   2 3
    6 X
  -----
   1 8   (6 X 3)
 1 2 0   (6 X 20)
 -----
 1 3 8
 -----
```

This is the meat in your sandwich!

Relate it directly to the areas of your grid.

We call this the skinny sandwich

```
   2 3
    6 X
  -----
 1 3 8
  -----
  1
```

When children are moving from grid to sandwich or compact method, lots of practice is needed and the progression should be:

- 1 digit number X 2 digit that cannot quickly be done mentally (involve some carrying over)
- 1 digit number X 3 or 4 digit number.

Remember that when you use big sandwich, you must get the children to write each step in brackets beside the product, otherwise they will forget their size because he place value is implicit. This happens even more when children move to small sandwich.
Note: some children will find this progression easy but when they try long multiplication, they will need to work on grid method for longer…

PROGRESSION THROUGH MULTIPLICATION: Models, Images & Written Methods

LONG MULTIPLICATION is when you multiply by anything more than a one digit number:

- Split screen teaching is needed to move from grid method to the expanded then the compact columnar method as with short multiplication. Make the links between these methods completely explicit and repeat split screen teaching as often as needed like this:

Grid Method

23 X 16 =

X	20	3
10	200	30
6	120	18

Big Sandwich:

```
   2 3
   1 6 X
  ------
   1 8   (6 X 3)
 1 2 0   (6 X 20)
   3 0   (10 X 3)
 2 0 0   (10 X 20)
  ------
 3 6 8
```

This is the really meaty bit!

Small Sandwich:

```
   2 3
   1 6 X
  ------
 1 3 8
   1
 2 3 0
  ------
 3 6 8
```

EP Consistency:

- Always write the equation horizontally before using a method.
- Children should estimate before calculating.
- When teaching the sandwich method, at EP we start with the smaller digit e.g. 6 X 3 then 6 X 20 and so on in the example above, but it is important that the children can see order is irrelevant as with the grid method.
- Adding up the areas of the grid can be done mentally, in two parts or as a full columnar addition.
- For the grid method, we don't need to use the 'one digit per square' rule in maths books but children do need enough space.

When children are moving from grid to sandwich or compact method, lots of practice is needed and the progression should be:

- 2 digit number X 2 digit that cannot quickly be done mentally
- 2 digit number X 3 digit number.

PROGRESSION THROUGH MULTIPLICATION

Mental Maths Supporting this Stage

Factual Fluency

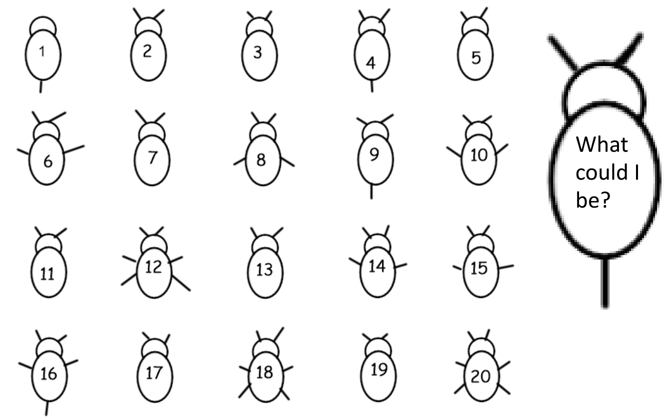
- Quickly recall factor pairs using times table facts.
- Identify common factors of two numbers.
- Identify common multiples.
- Learn the meaning of: prime number, prime factors and composite (non-prime) numbers.
- Recall prime numbers to at least 20.
- Recall square numbers to 12^2 and their square roots
- Double and halve numbers to 1,000 then 10,000.

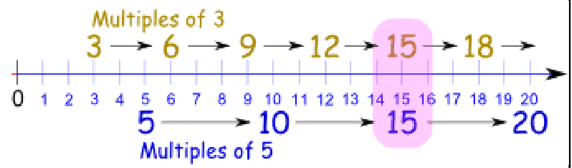
Mental Strategies

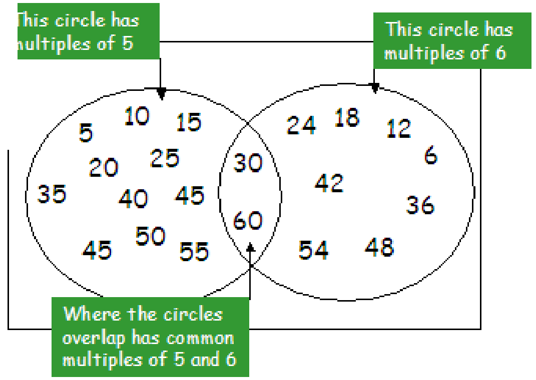
- Establish whether a number up to 100 and then 1,000 is prime using divisibility tests
- Find the prime factors of numbers to 100.
- Recognise and use square numbers and cube numbers with the notation for squared ^{2} and cubed^3.
- Recognise and use square roots and the symbol √

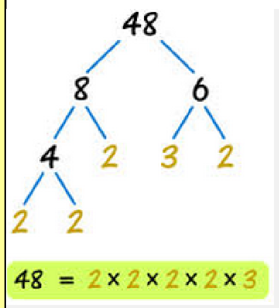
Models, Images & Written Methods

- Use factor bugs as a visual way of understanding factors. Explore what a prime number is and then use 'splits' to find prime factors of numbers:









- Using this image of 'splits' or factorisation, teach children that a number could be the product of more than two factors as with 48 in this image of factorising or 24 = 2 X 2 X 6 (this has not been boiled down to its prime factors but is still true!).

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PROGRESSION THROUGH MULTIPLICATION

Mental Maths Supporting this Stage

Factual fluency

- Multiply and **divide** whole numbers and decimal numbers by 10, 100, 1,000 and 10,000 rapidly. Know that the digits move one, two, three or four places to the left or right relative to the decimal point and zero is used as a place holder.

Mental Strategies

- Add up amounts of money, and measures, converting between £ and p.
- Convert larger to smaller units of measurement using decimals to one place, e.g. change 2.6kg to 2,600g, 3.5cm to 35mm, 1.2m to 120cm
- Use doubling and halving to make calculations easier to manage eg:
  12 X 1.35 = 6 X 2.7
  14 X 24 = 7 X 48
- Use nearly numbers to make tricky calculations easier to manage e.g:
  19 X 12 = (20 X 12) – 12
- Multiply by 25 or 50 by using equivalent calculations:
  48 X 25 = 48 X 100 ÷ 4
  32 X 50 = 32 X 100 ÷ 2
  Work towards doing this rapidly as part of **factual fluency.**

Models, Images & Written Methods

- Explore division of numbers by 10, 100 and 1,000 using place value sliders as shown above.
- Introduce problems that require multiplying an amount of money by a one digit number (going back to short multiplication with decimals):
  Judge whether your children have a deep enough understanding of decimals to represent money as below in the 3rd example, or if converting to pence is most straightforward. Make similar choices when multiplying measures such as cm and m, l and ml etc.

£1.35 X 6 =

X	100p	30p	5p
6	600p	180p	30p = 810p = £8.10

or

X	£1	30p	5p
6	£6	£1.80	30p = £8.10

Or

£1.35 X 6 =

X	£1	£0.30	£0.05
6	£6	£1.80	£0.30 = £8.10

Can be taught alongside the big then small sandwich:

```
 135
   6 X
 ----
  30   (6 X 5p)
 180   (6 X 30p)
 600   (6 X 100p)
 ----
 810   so the answer is £8.10
 1
```

```
 £
 1.35
    6 X
 -----
 8.10
 -----
 2 3
```

<table>
<tr><th></th><th colspan="2">PROGRESSION THROUGH DIVISON</th></tr>
<tr><th></th><th>Mental Maths Supporting this Stage</th><th>Models, Images & Written Methods</th></tr>
<tr><td></td><td>
Factual Fluency

Begin to share out objects into two groups
Count back in 2s, 5s and 10s. Don’t stop after 20 or 50 or 100!
Learn and recall doubles and halves of all numbers to 10 then 20.
Know odd and even numbers to 10, 20 then up to 100 in KS1. (Numicon!)

Mental Strategies

Find corresponding halves for doubles of numbers to 20 e.g. half 26 by knowing double 13.

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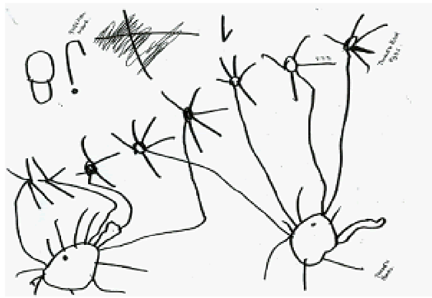
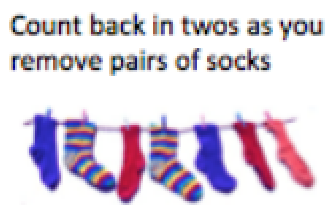
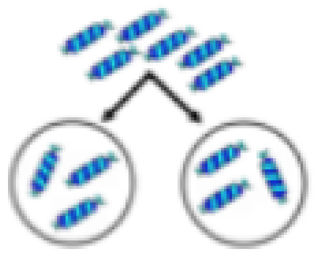

A 4-year-old’s representation of ‘Can 8 be shared equally between two?’
Division requires the development of the ideas of ‘sharing’ and ‘grouping’. Sharing is linked to fractions and crops up naturally in all sorts of situations at school and home.
Grouping can be developed in parallel with multiplicative thinking. The use of arrays is therefore a key tool because arrays represent both multiplication and division in one, and lead on to grid method multiplication and bus stop division.

Early on, encourage equal sharing of objects (making links with fractions).




</td></tr>
</table>

PROGRESSION THROUGH DIVISION

Mental Maths Supporting this Stage

Factual Fluency

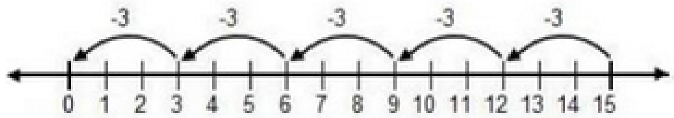
- Recall half of even numbers to 40.
- Learn the division facts alongside learning these multiplication facts (see multiplication progression).

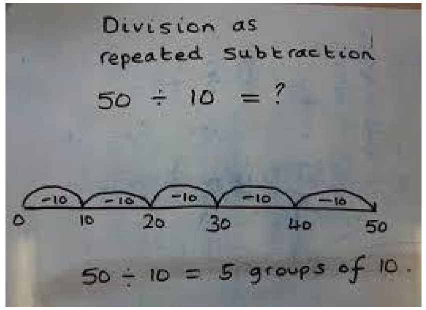
Mental Strategies

- Find half of any multiple of 10 up to 100.
- Use knowledge that halving is the inverse of doubling and equivalent to dividing by 2.
- When halving, partition and halve the tens and ones separately then recombine.
- Find the total number of objects when they are organised into groups of 2, 3, 5 or 10.
- Notice that to divide by 10, shift the digits to the right one place e.g. 40 ÷ 10 = 4
- Finding remainders of 1 first, then greater remainders by using 'inside out times tables'.

Models, Images & Written Methods

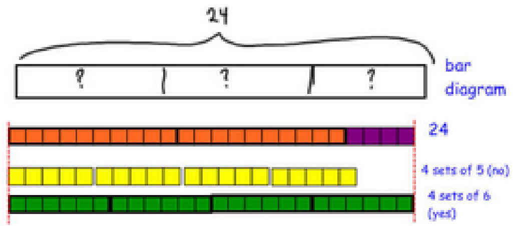
- Children will understand division as repeated subtraction, the inverse of multiplying by repeated addition. Use the same model of the number line, counting back and emphasising that you are finding out how many 'equal groups of' the number fit inside:



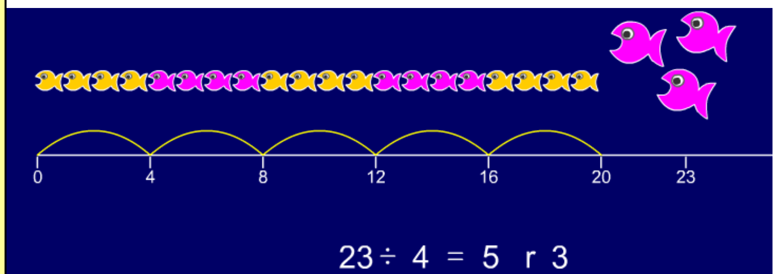


Note: **chunking backwards** using subtraction is returned to as a method of long division later in this progression but **chunking forwards** is easier for mental division using tables.

- Use bar models alongside cuisenaire to help children visualise how many 'groups of' a number fit inside the larger number. This works well alongside chunking using a number line.



- When the children have a very secure understanding of finding how many 'groups of', **chunking forward** will help the children use their times table facts to solve problems.



ITP Grouping (search online for this) is brilliant for showing this model, allowing you to group and modelling automatically on a number line.

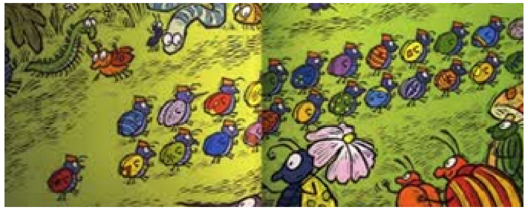
PROGRESSION THROUGH DIVISION: Models, Images & Written Methods

- Practice mental division as 'inside out times tables' using empty boxes and fact families. This should continue to Y6 and beyond!

☐ x 5 = 20   3 x △ = 18   ☐ x O = 32   26 ÷ 2 = ☐   24 ÷ △ = 12   ☐ ÷ 10 = 8

"How many groups of 5 in 20?"   "3 groups of what make 18?"   "how many 2s are in 26?"   "12 groups of what are in 24?"

- Explore the concept of remainders using both number line chunking and arrays (can use Numicon). The book 'Remainder of One' uses arrays to explore remainders.

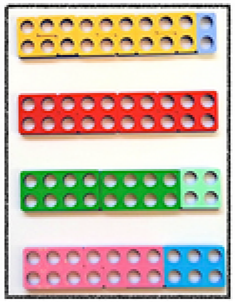


20 ÷ 3 = 6r2

20 ÷ 5 = 4

20 ÷ 8 = 2r4

20 ÷ 7 = 2r6



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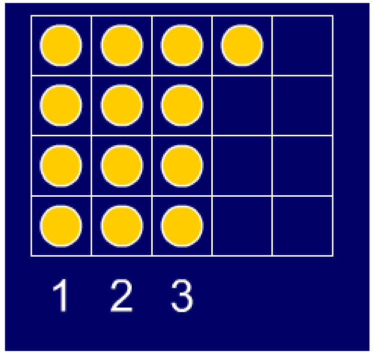
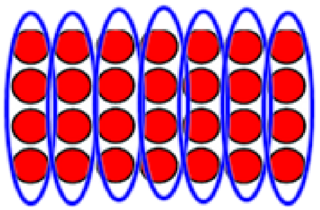


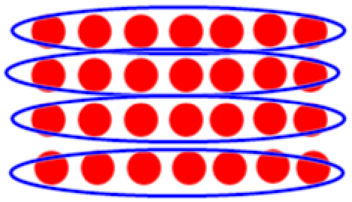
Use ITP Remainders After Division (search online for this) to model counting 'groups of' in an array as well remainders.

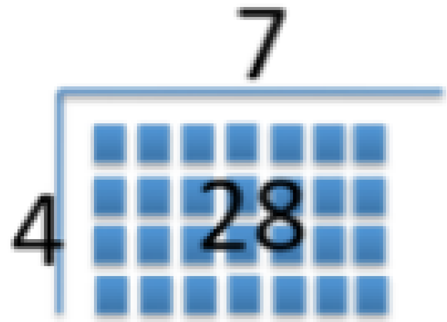
- Now you can start to use the image of an array to move towards formal written method of division. Bus stop short division (dividing by a 1 digit number) can now be directly linked to the grid method image:



7 groups of 4

(this can also show 4 groups of 7)



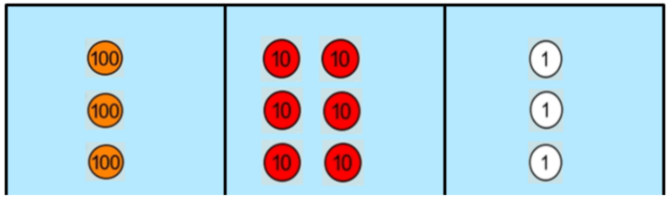


PROGRESSION THROUGH DIVISION: Models, Images & Written Methods

Our aim is for children to teach the formal method conceptually so throughout this progression, work on chunking forwards with mental jottings on a number line to fully develop children's understanding of division. e.g. to find out how many 50p in £5, count forward in 50ps.

$363 \div 3 =$

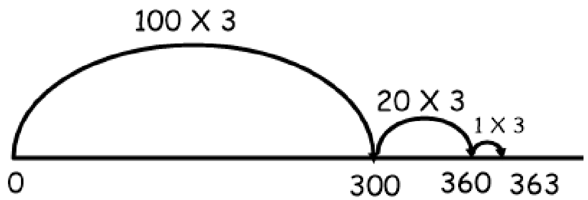
$$\begin{array}{r|l} & 1\ 2\ 1 \\ \hline 3 & 3\ 6\ 3 \end{array}$$



- Use place value counters to model short division (bus stop) first without and then involving remainders as follows:

Refer to making groups of 3 using the place value counters and show the formal 'bus stop' algorithm above place value counters which you arrange in groups of 3.

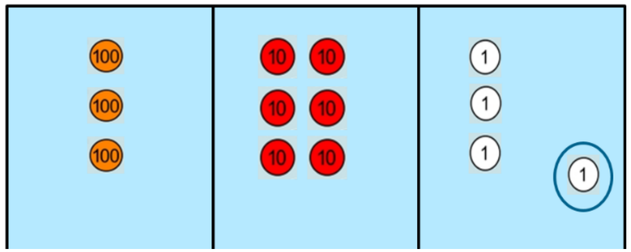
Using place value counters is an excellent way of representing chunking/grouping by finding that we can get 100 groups of 3 out of 363, then 60 groups, then 1 group. It could be jotted on a number line to further develop conceptual understanding:



- Next, show the same calculation but with a remainder of 1:

$$\begin{array}{r|l} & 1\ 2\ 1\ r\ 1 \\ \hline 3 & 3\ 6\ 3 \end{array}$$

Top tip: divide by familiar times tables and those that need practice!
Don't divide by 2 (halve it!) or 4 (halve it and halve it again!).
Arguably, you shouldn't use bus stop to divide by 5 either because you can divide by 10 then halve it!



PROGRESSION THROUGH DIVISION

Mental Maths

Factual Fluency

- Use times tables inside out to find division facts (fact families) up to 144 ÷ 12

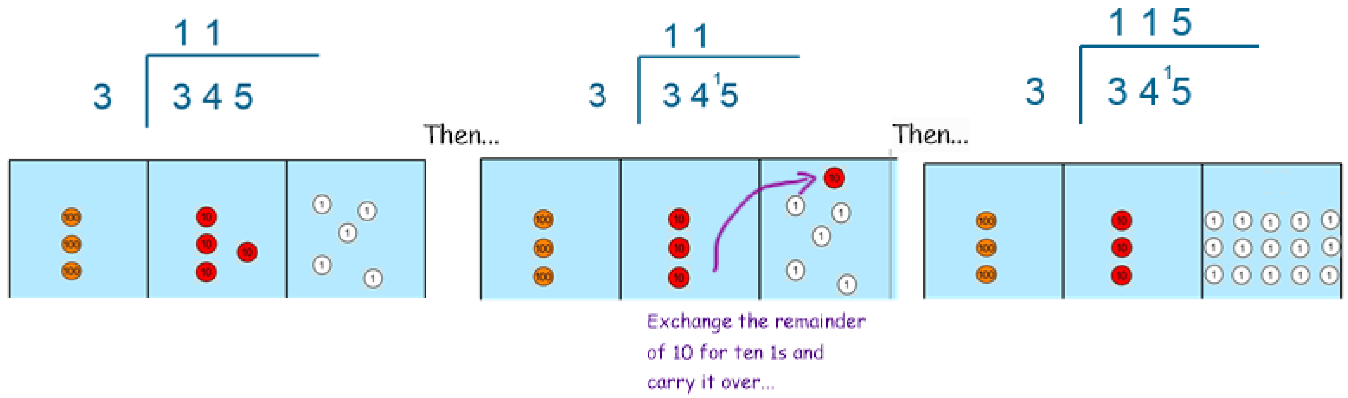
Mental Strategies

- Notice that when dividing by 100, the digits shift to the right two places.
- Use times table facts inside out, to find related division facts by adjusting place value e.g. 14 ÷ 7 = 2 so 140 ÷ 7 = 20
- Relate dividing to finding fractions of numbers ie. ÷2 is finding half, ÷3 is finding a third, ÷4 is finding quarter and so on. Important: teach ÷10 is finding a tenth (slide the digits!)
- Understand that the biggest remainder one can have is the integer before our divisor e.g. when dividing by 8, our biggest remainder could be 7.

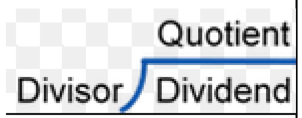
Models, Images & Written Methods

- Next, progress to using short division where there is one remainder within the calculation. Model this using place value counters:

345 ÷ 3 =



- Plan your calculations with remainders in mind, following this progression:
  - No remainders
  - Remainder after the ones have been divided. Remainder of 1, then 2 and other numbers.
  - One remainder within the calculation but a whole number answer.
  - Remainders within the calculation and left over at the end.
  - What happens when the first digit we come to (most significant digit) is smaller than our divisor?
  - What happens when there is a zero in our dividend?
  - What if our dividend is a decimal e.g. dividing £4.56 by 3? Ensure the decimal place in the quotient is always lined up in the correct place.

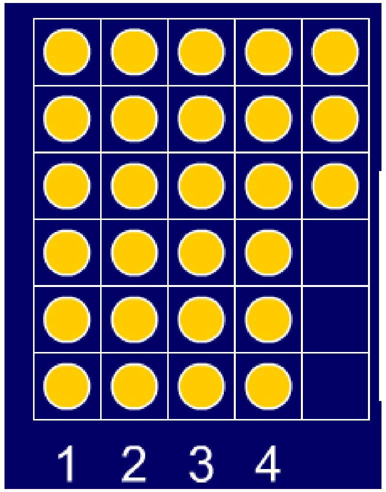


PROGRESSION THROUGH DIVISION: Models, Images & Written Methods

- Children need to be able to decide what to do after division and round up or down accordingly. For example 62 ÷ 8 is 7 remainder 6, but whether the answer should be rounded up to 8 or rounded down to 7 depends on the context
  - I have 62p. Sweets are 8p each. How many can I buy? Answer: 7 (the remaining 6p is not enough to buy another sweet)
  - Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed? Answer: 8 (the remaining 6 apples still need to be placed into a box)
- Next, teach children how to express remainders as a vulgar fraction, ie. When we calculate 27 ÷ 6 we have a remainder of 3. 3 is half of our 'group' of 6 and therefore we have 3/6 or ½ a group left.

  This can be modelled with a number line, showing the remaining jump as half the size of the others, or using an array, which clearly shows 4 full groups of 6 and half a group left.

- Teach the children that whatever our divisor is (our group size), this can become the denominator and our remaining whole number is our numerator. Practice expressing remainders in this way. Note: children at this stage must be fluent in their understanding of fractions **as** numbers. Stick to finding ½ and equivalent remainders, and ¼ or ¾ or equivalent. Find 1/3 but do not find decimal equivalent until children have explored remainders that have one or two place equivalents.



- Explore why my calculator gives the answer 4.5 when calculating 27 ÷ 6 (5/10 = 0.5). This is the introduction to finding decimal remainders.
- Once children have understanding of remainders as decimal fractions, they can be taught to find decimal remainders using the short division algorithm, placing a decimal point and zeros next to the dividend until the remainder is complete or recurs:

$$142 \div 4 = 35{\cdot}5$$

r2

2/4 = 1/2 = 0.5

$$\begin{array}{r} 0\,3\,5{\cdot}5 \\ 4\,\overline{)\,1\,{}^{1}4\,{}^{2}2{\cdot}{}^{2}0} \end{array}$$

$$1089 \div 8 =$$

$$\begin{array}{r} 1\,3\,6\,.\,1\,2\,5 \\ 8\,\overline{)\,1\,{}^{1}0\,{}^{2}8\,{}^{4}9\,.\,{}^{1}0\,{}^{2}0\,{}^{4}0} \end{array}$$

Children must be taught to decide when an answer requires a remainder as a whole number (individual items being grouped), a vulgar fraction (a continuous whole divided) or decimal fraction (measures or money).

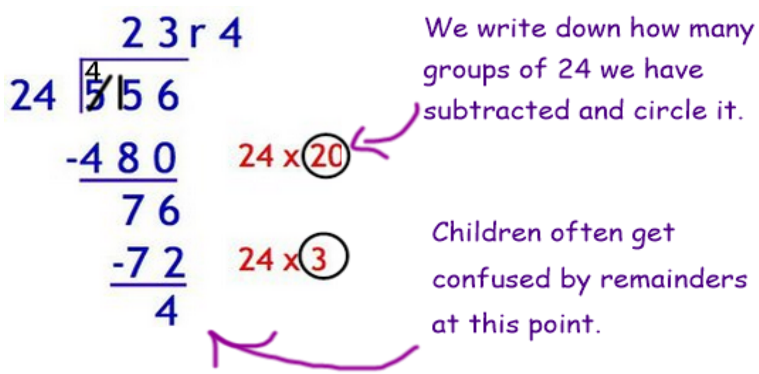
PROGRESSION THROUGH DIVISION: Models, Images & Written Methods

LONG DIVISION: where the divisor is more than a one digit number.

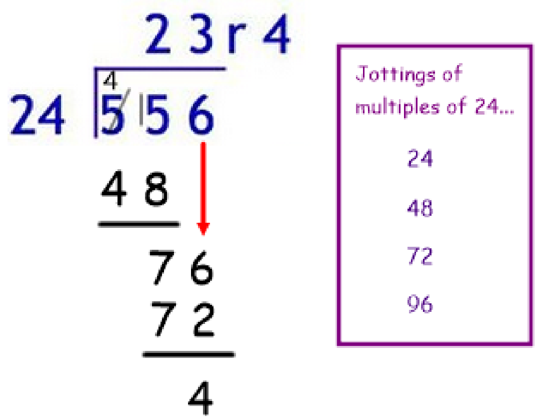
- As children's mental division progresses, they must be taught to chunk forwards (as shown further back in this document). For example, to find out how many comics costing £1.50 you can buy with £9, you would not need a written method of long division, but rather count up in £1.50 until you reach £9. This mental agility must be developed with lots of practical examples and practice.

  ** Unless a child is competent at times tables, has a firm grasp of place value and can do decomposition subtraction with ease, do not expect them to be able to do 'chunking' or any 3 digit by 2 digit division **
- First teach long division using this expanded method, often called chunking. The key here is to explain you are finding out how many 'groups of' the divisor are inside the dividend… or how many "chunks" of the divisor you can take away. Emphasise that we could take away groups of the number one at a time but that we would be here all day! Maybe we could try taking away 10 chunks, or 20 chunks, or 100! The freedom of choosing your chunk size can be challenging so plan your example carefully. 556 ÷ 24 =

CHUNKING



FORMAL ALGORITHM FOR LONG DIVISION



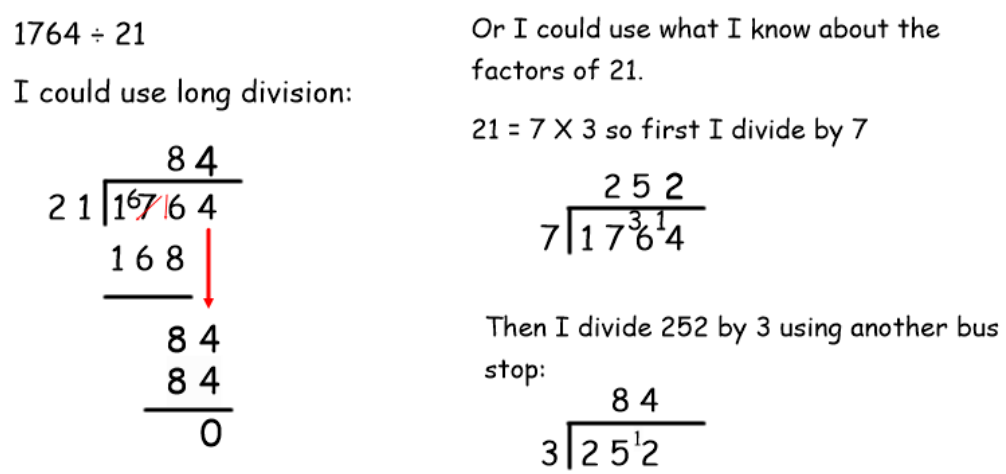
- **On the left is the chunking method of long division and beside it on the right, the formal long division method as shown in the National Curriculum appendix 2014. There is a progression to be made between them and both can support children finding decimal remainders as with short division. Both require mental multiplication of the divisor to support the calculation.**

PROGRESSION THROUGH DIVISION: Models, Images & Written Methods

- Progression through long multiplication:
  - Encouraging children to take bigger chunks away if they are chunking! 50x in one go rather than 10x repeatedly
  - Working with bigger dividends
  - Remainders
  - Extend to decimals with up to two decimal places. Children should know that decimal points line up under each other

Written and Mental Alternatives To Long Division

- Encourage the children to use factors of the divisor to turn the division into a more manageable short division problem:



- Using halving is another great way to avoid long division and solve a problem mentally e.g. 248 ÷ 16 = 124 ÷ 8 Investigate more problems such as these with the children to support their understanding of scaling up and down.